The Syllogistic Reasoning Task:
Reasoning Principles and Heuristic Strategies in Modeling Human Clusters

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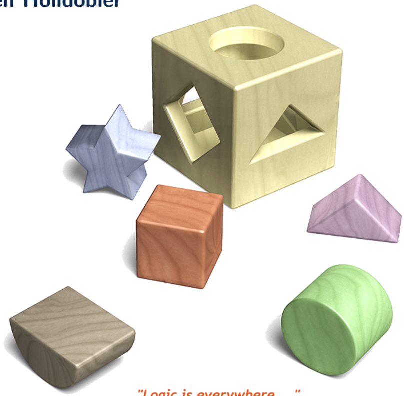


"Logic is everywhere ..."

Modelling Human Reasoning

- Motivation: understanding human reasoning on a high level (psychology)
- Goal: provide a formalism that can model episodes of human reasoning
- In particular, predict the outcomes of psychological experiments, e.g.
  - Wason's selection task
  - Byrne's suppression task
  - Syllogistic reasoning
- Why syllogisms?
  - Well known and researched, exist since Aristotle
  - Not trivial, but also not too complex as a task
- Meta-analysis on syllogistic reasoning: Khemlani and Johnson-Laird (2012)
  - Data from six psychological studies (156 participants)
  - Comparison of twelve cognitive theories

The Syllogistic Reasoning Task

- What conclusion follows from two given premises?
  E.g. *All artists are bakers.*
  Some chemists are not bakers.
- Opinions vary:
  - Some chemists are not artists (Oca)
    - FOL, Humans, Mental Models, Verbal Models, PSYCOP
  - No valid conclusion (NVC)
    - Humans, Mental Models, Verbal Models
  - Other answers (not significant)
- FOL cannot model human syllogistic reasoning adequately
- *The* human reasoner might not exist (Oca vs. NVC)

Formalization of the Syllogistic Reasoning Task

- The premises of a syllogism are categorized by two criteria:

	affirmative	negative
$\forall$	A	E
$\exists$	I	O

Mood (type of the quantifier)

	b-c	c-b
a-b	1	3
b-a	4	2

Figure (arrangement of the terms)

- "All a are b, some c are not b" is AO3
- There are $4 \times 4 \times 4 = 64$ logically distinct premise pairs
- There are $4 \times 2 + 1 = 9$ possible answers
- Data: 64×9 matrix of answer vectors for each premise pair
- Evaluation: match of theory's prediction to participants' data

Three-valued Contextual Programs

- We use the three-valued logic defined by Łukasiewicz ($\top, \bot, \mathsf{U}$)
- We consider datalogic programs with the following types of clauses:

Rule: $A \leftarrow B_1 \wedge \cdots \wedge B_m \wedge \neg B_{m+1} \wedge \cdots \wedge \neg B_{m+n}.$

Fact: $A \leftarrow \top.$

Negative assumption: $A \leftarrow \bot.$

Unknown assumption: $A \leftarrow \mathsf{U}.$

(A and B_i are atoms)

- We denote the three-valued interpretation of a program $\mathcal{P}$ as $I = \langle I^\top, I^\bot \rangle$
- The context operator [Dietz Saldanha et al., 2017] may be applied to atoms:

$$\mathrm{ctxt}(A) = \begin{cases} \top & I(A) = \top, \\ \bot & \textit{otherwise}. \end{cases}$$

The Weak Completion Semantics

- For a logic program $\mathcal{P}$, the *weak completion*, $wc\mathcal{P}$, is obtained by
  1. For each atom A in $g\mathcal{P}$, replace all clauses of the form $A \leftarrow \text{body}_1, \ldots A \leftarrow \text{body}_n$ by $A \leftarrow \text{body}_1 \vee \cdots \vee \text{body}_n$
  2. Replace all occurrences of $\leftarrow$ by $\leftrightarrow$
- We are interested in minimal models of $wc\mathcal{P}$
- We compute them with the $\Phi_\mathcal{P}$ operator [Stenning and van Lambalgen, 2008]

  For a three-valued interpretation $I = \langle I^\top, I^\bot \rangle$, $\Phi_\mathcal{P}(I) = \langle J^\top, J^\bot \rangle$, where

$$
\begin{array}{lcl}
J^\top & = & \{A \mid A \leftarrow Body \in g\mathcal{P} \text{ and } Body \text{ is } true \text{ under } \langle I^\top, I^\bot \rangle\} \\
J^\bot & = & \{A \mid A \leftarrow Body \in g\mathcal{P} \text{ and} \\
& & \quad Body \text{ is } false \text{ under } \langle I^\top, I^\bot \rangle \text{ for all } A \leftarrow Body \in g\mathcal{P}\}
\end{array}
$$

- The least fixed point of $\Phi_\mathcal{P}$ is a minimal model of $wc\mathcal{P}$ ($\mathcal{M}_\mathcal{P}$)

Reasoning under the WCS

- Foundations by Hölldobler and Ramli [2009]
- Application to many episodes of human reasoning:
  - Wason's selection task [Dietz et al., 2012]
  - Byrne's suppresion task [Dietz et al., 2013]
  - The Belief-bias effect [Pereira et al., 2014a,b]
  - Reasoning about conditionals [Dietz and Hölldobler, 2015, Dietz et al., 2015b]
  - Spatial reasoning [Dietz et al., 2015a]
  - Syllogistic reasoning [Dietz, 2015]
- Recent application to syllogisms [da Costa et al., 2017]
  - Introduction of principles of reasoning
  - Still assuming one human reasoner

Basic Principles of Human Reasoning

- Encoding of premises motivated by findings from literature [da Costa et al., 2017]
- Basic principles are assumed to be used by all reasoners
- Quantified statements as conditionals (all moods)
  $z(X) \leftarrow y(X).$
- Licenses for inferences (all moods)
  $z(X) \leftarrow y(X) \wedge \neg ab_{yz}(X).$
- Existential import (all moods)
  $y(o_1) \leftarrow \top.$
  $ab_{yz}(o_1) \leftarrow \bot.$
- Unknown generalization (existential moods)
  $y(o_2) \leftarrow \top.$
- No refutation by counterexample (universal moods)
  $ab_{yz}(X) \leftarrow \bot.$

Advanced Principles of Human Reasoning

- Advanced principles are not necessarily applied by all reasoners
- Search for alternative conclusions (abduction)
  - Humans might not be satisfied if they come to *no valid conclusion*
  - Some may test additional conclusions
- Converse premise (moods I and E)
  - The premises *some a are b* and *some b are a* are logically equivalent
  - The corresponding converse premise may be added to the encoding as well
- Deliberate generalization (mood I)
  - Unknown generalization allows inference only for one object
  - Humans might reason a bit more relaxed
- Contraposition (mood A)
  - In FOL, “a implies b” is equivalent to “not b implies not a”

Logic Program of the Syllogism AO3 (basic principles)

- All a are b:

$b(X) \leftarrow a(X) \wedge \neg ab_{ab}(X).$	conditional, licenses
$a(o_1) \leftarrow \top.$	exImport
$ab_{ab}(o_1) \leftarrow \bot.$	exImport, licenses
$ab_{ab}(X) \leftarrow \bot.$	no refutation, licenses

- Some c are not b:

$b'(X) \leftarrow c(X) \wedge \neg ab_{cnb}(X).$	conditional, licenses
$c(o_2) \leftarrow \top.$	exImport
$ab_{cnb}(o_2) \leftarrow \bot.$	exImport, licenses
$c(o_3) \leftarrow \top.$	unknownGen
$b(X) \leftarrow \neg b'(X) \wedge \neg ab_{nbb}(X).$	transformation, licenses
$ab_{nbb}(o_2) \leftarrow \bot.$	doubleNeg, licenses
$ab_{nbb}(o_3) \leftarrow \bot.$	doubleNeg, licenses

Model and Conclusions of AO3

- Conclusions are entailed from the least fixed point of $\Phi_{\mathcal{P}}$ by logic formulas
- Least fixed point of $\Phi_{\mathcal{P}_{\text{AO3,basic}}}$ (AO3 with basic principles):

$$\mathcal{M}_{\mathcal{P}_{\text{AO3,basic}}} = \langle \{a(o_1), b(o_1), c(o_2), c(o_3), b'(o_2)\}, \{ab_{ab}(o_1), ab_{ab}(o_2), ab_{ab}(o_3), ab_{cnb}(o_2), ab_{nbb}(o_2), ab_{nbb}(o_3)\}\rangle$$

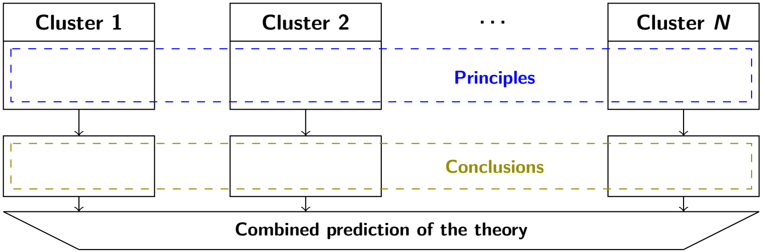
- No valid conclusion (NVC) is entailed
- If the encoding of the contraposition principle is added, we have:

$$\mathcal{M}_{\mathcal{P}_{\text{AO3,cp}}} = \langle \{a(o_1), ab_{ab}(o_2), b(o_1), c(o_2), c(o_3), a'(o_2), b'(o_2)\}, \{a(o_2), ab_{ab}(o_1), ab_{ab}(o_3), ab_{cnb}(o_2), ab_{nba}(o_1), ab_{nba}(o_2), ab_{nba}(o_3), ab_{nbb}(o_2), ab_{nbb}(o_3), b(o_2), a'(o_1)\}\rangle$$

- From this model "Some c are not a" (Oca) follows

Clusters of Reasoners

- **Prediction for the syllogism AO3 by the WCS:**
  - ***Some c are not a*** **(Oca) if the contraposition principle is applied**
  - ***No valid conclusion*** **(NVC) if it is not applied**
- **Participants' answers: both Oca (40 %) and NVC (20 %)**
- **We model clusters that differ in the application of advanced principles:**



Heuristic Strategies

- Atmosphere [Woodworth and Sells, 1935]
  - The premises create an atmosphere depending on their mood
  - A negative atmosphere makes a positive conclusion unlikely
  - An existential atmosphere makes a universal conclusion unlikely
- Matching [Wetherick and Gilhooly, 1995]
  - Defines an order of "conservativity" on moods: $E > O = I > A$
  - More conservative $\widehat{=}$ assertion about a smaller set of entities
  - A conclusion cannot be less conservative than any of the premises
- Biased conclusions in premises with figure 1
  - Recall the arrangement of terms in figure 1: (a-b, b-c)
  - Participants almost always answer Xac and never Xca for a mood X
  - Other answers are rare, too

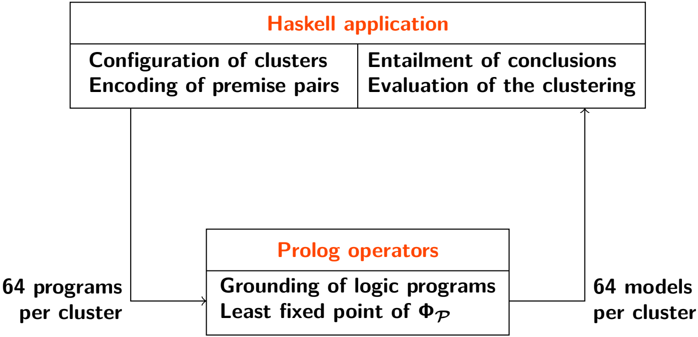
A Clustering Approach

- We model the following clusters of reasoners (in terms of used principles):
  1. Basic and converse interpretation (mood I)
  2. Basic, converse interpretation (mood I), and deliberate generalization
  3. Basic, converse interpretation (moods I and E) and contraposition
- We filter the predictions by the two heuristic strategies:
  1. Matching strategy
  2. Biased conclusions in premises with figure 1
- We predict 32 of 64 premise pairs without error

PSYCOP	Verbal Models	Mental Models	WCS (2017)	WCS (clusters)
77 %	84 %	78 % / 90 %	89 %	92 %

The WCS vs. the best cognitive theories of [Khemlani and Johnson-Laird, 2012].

Implementation



Conclusion

- The syllogistic reasoning task is an important episode of human reasoning
- FOL and well-known cognitive theories fail to model it adequately
- Our approach:
  - Identification of principles of reasoning
  - Clusters of humans in terms of used principles
  - Reasoning under the Weak Completion Semantics
  - Integration of heuristic strategies
- Future work: better clustering, quantitative predictions

Thank you for your attention. Questions?

Technical Aspects of Encoding Principles

- Negated atoms cannot be the head of a logic program clause
- The basic principles alone cannot distinguish between positive and negative moods
- Negation by transformation

  $z'(X) \leftarrow y(X) \wedge \neg ab_{nyz}(X).$
  $z(X) \leftarrow \neg z'(X) \wedge \neg ab_{nzz}(X).$

- Under the WCS, it is possible to derive knowledge through double negation

$$\mathcal{P}_{\text{doubleNeg}} = \{b \leftarrow \neg a, c \leftarrow \neg b, a \leftarrow \top\}$$
$$\mathcal{M}_{\mathcal{P}_{\text{doubleNeg}}} = \langle\{a, c\}, \{b\}\rangle$$

- No derivation through double negation

  $ab_{nzz}(o_1) \leftarrow \bot.$
  $ab_{nzz}(o_2) \leftarrow \bot.$ (if *unknown generalization* is used as well)

Entailment of Conclusions (Deduction and Abduction)

- Deduction (forward reasoning): conclusions follow from the least fixed point

  Ayz $\exists X(\mathcal{P} \models_{wcs} y(X)) \wedge \forall X(\mathcal{P} \models_{wcs} y(X) \rightarrow \mathcal{P} \models_{wcs} z(X))$

  Eyz $\exists X(\mathcal{P} \models_{wcs} y(X)) \wedge \forall X(\mathcal{P} \models_{wcs} y(X) \rightarrow \mathcal{P} \models_{wcs} \neg z(X))$

  Iyz $\exists X(\mathcal{P} \models_{wcs} y(X) \wedge z(X)) \wedge \exists X(\mathcal{P} \models_{wcs} y(X) \wedge \mathcal{P} \not\models_{wcs} z(X)) \wedge \exists X(\mathcal{P} \models_{wcs} z(X) \wedge \mathcal{P} \not\models_{wcs} y(X))$[1]

  Oyz $\exists X(\mathcal{P} \models_{wcs} y(X) \wedge \neg z(X)) \wedge \exists X(\mathcal{P} \models_{wcs} y(X) \wedge \mathcal{P} \not\models_{wcs} \neg z(X))$

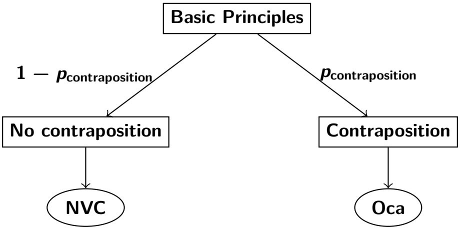
  NVC None of the above conclusions is entailed

- Abduction (backward reasoning): conclusions are computed by *abductive frameworks* that sequentially try to explain observations
  - Observations are facts that are not the head of a rule (existential import)
  - If they can be explained, new conclusions may follow as well
  - How are explanations defined? See the paper

[1] This third part of the conjunction is only used if the principle *converse interpretation* is applied.

Multinomial Processing Trees (MPT)

- Origin: Riefer and Batchelder (1988)
- Cognitive processes as inner nodes
- Response categories (here: conclusions) as leaves
- Probabilistic transition between processes (learnt parameters)



MPT for the syllogism AO3

Quantitative Evaluation

- We use the evaluation method of [Khemlani and Johnson-Laird, 2012]
- An answer is significant if it is given by more than 16 % of the participants
- Answers and predictions are encoded as vectors $\in \{0,1\}^9$
- For each premise pair, the quality of the prediction is as follows:

$$\text{score}(v_{answ}, v_{pred}) = \frac{1}{9} \times \sum_{i=1}^{9} [v_{answ}(i) = v_{pred}(i)]$$

- The overall quality of a theory is the average over all premise pairs

PSYCOP	Verbal Models	Mental Models	WCS	WCS (clusters)
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The WCS vs. the best cognitive theories of [Khemlani and Johnson-Laird, 2012].

Implementation

- Given a clustering approach, compute
  - The theory's predictions for all 64 premise pairs
  - The quantitative evaluation with respect to the participants' answers
- Component 1: implementation of $\Phi_{\mathcal{P}}$
  - Encoding of logic programs directly in Prolog
  - Prolog implementation grounds these programs and computes $\Psi_{\mathcal{P}}$
- Component 2: computing the predictions and score for a clustering
  - Representation of principles and clusters as Haskell datataypes
  - Haskell implementation generates the program for each cluster and premise pair
  - Prolog implementation is called to obtain the least fixed point
  - Conclusions are derived from that model and combined
  - Option to compute the fit to the participants' answers
  - Option to generate a MPT for each premise pair

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- Converse premise (moods I and E)
  - The premises *some a are b* and *some b are a* are logically equivalent
  - The corresponding converse premise may be added to the encoding as well
- Deliberate generalization (mood I)
  $ab_{yz}(X) \leftarrow \text{ctxt}(z'(X)).$
  $ab_{yz}(o_2) \leftarrow \text{U}.$
- Contraposition (mood A)
  $y'(X) \leftarrow \neg z(X) \wedge \neg ab_{nzy}(X).$
  $y(X) \leftarrow \neg y'(X) \wedge \neg ab_{nyy}(X).$
  $ab_{nzy}(X) \leftarrow \bot.$

Bibliography I

Ana Oliveira da Costa, Emmanuelle-Anna Dietz Saldanha, and Steffen Hölldobler. Monadic reasoning using weak completion semantics. 2017.

Emanuelle-Anna Dietz. A computational logic approach to syllogisms in human reasoning. In *Bridging@ CADE*, pages 17–31, 2015.

Emmanuelle-Anna Dietz and Steffen Hölldobler. A new computational logic approach to reason with conditionals. In *International Conference on Logic Programming and Nonmonotonic Reasoning*, pages 265–278. Springer, 2015.

Emmanuelle-Anna Dietz, Steffen Hölldobler, and Marco Ragni. A computational logic approach to the suppression task. In *CogSci*, 2012.

Emmanuelle-Anna Dietz, Steffen Hölldobler, and Marco Ragni. A computational logic approach to the abstract and the social case of the selection task. In *Proceedings of the 11th International Symposium on Logical Formalizations of Commonsense Reasoning, COMMONSENSE*, 2013.

Emmanuelle-Anna Dietz, Steffen Hölldobler, and Raphael Höps. A computational logic approach to human spatial reasoning. In *Computational Intelligence, 2015 IEEE Symposium Series on*, pages 1627–1634. IEEE, 2015a.

Emmanuelle-Anna Dietz, Steffen Hölldobler, and Luís Moniz Pereira. On conditionals. In *GCAI*, pages 79–92, 2015b.

Bibliography II

Emmanuelle-Anna Dietz Saldanha, Steffen Hölldobler, and Luís Moniz Pereira. Contextual reasoning: Usually birds can abductively fly. 2017.

Steffen Hölldobler and Carroline Dewi Puspa Kencana Ramli. Logic programs under three-valued łukasiewicz semantics. In *International Conference on Logic Programming*, pages 464–478. Springer, 2009.

Sangeet Khemlani and P. N. Johnson-Laird. Psychological bulletin theories of the syllogism: A meta-analysis. 2012.

Luís Moniz Pereira, Emmanuelle-Anna Dietz, and Steffen Hölldobler. An abductive reasoning approach to the belief bias effect. In *KR*, 2014a.

Luis Moniz Pereira, Emmanuelle-Anna Dietz, and Steffen Hölldobler. Contextual abductive reasoning with side-effects. *Theory and Practice of Logic Programming*, 14 (4-5):633–648, 2014b.

David M Riefer and William H Batchelder. Multinomial modeling and the measurement of cognitive processes. *Psychological Review*, 95(3):318–339, 1988.

Keith Stenning and Michiel van Lambalgen. Human reasoning and cognitive science. a bradford book. In *Cambridge MA*. MIT Press, 2008.

N. E. Wetherick and K. J. Gilhooly. ‘atmosphere’, matching, and logic in syllogistic reasoning. *Current Psychology*, 14(3):169–178, 1995.

R. S. Woodworth and S. B. Sells. An atmosphere effect in formal syllogistic reasoning. *Journal of Experimental Psychology*, 18(4):451, 1935.