

# Techniques for Efficient Lazy-Grounding ASP Solving

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# Introduction

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# Motivation

- Answer-Set Programming (ASP): well-established KR formalism.
- Rule-based, nonmonotonic, expressive.
- Traditional ASP solving two-phased: **ground** and **solve**.
- Problem: grounding of **exponential size** in general and practice.

⇒ **Grounding bottleneck** of ASP.

## Example

*dom(1). dom(2). ... dom(20).*

*sel(X) ← dom(X), not nsel(X).*

*nsel(X) ← dom(X), not sel(X).*

*← sel(Y), sel(X), X ≠ Y.*

*p(X<sub>1</sub>, X<sub>2</sub>, X<sub>3</sub>, X<sub>4</sub>, X<sub>5</sub>, X<sub>6</sub>) ← sel(X<sub>1</sub>), sel(X<sub>2</sub>), sel(X<sub>3</sub>),  
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## Example

$dom(1). dom(2). \dots dom(20).$

$sel(X) \leftarrow dom(X), not nsel(X).$

$nsel(X) \leftarrow dom(X), not sel(X).$

$\leftarrow sel(Y), sel(X), X \neq Y.$

$p(X_1, X_2, X_3, X_4, X_5, X_6) \leftarrow sel(X_1), sel(X_2), sel(X_3),$   
 $sel(X_4), sel(X_5), sel(X_6).$

# Lazy-Grounding ASP Solving

- Lazy-Grounding: interleave grounding and solving  
⇒ **Avoid grounding bottleneck.**
  - Based on **computation sequences.**
  - Existing solvers: Gasp, AsPeRiX, Omega.
  - No strong **search space reducing techniques**  
(like conflict-driven learning, backjumping, heuristics, etc).
  - Grounding strongly-coupled to search (grounder backtracks).
- ⇒ Bad search performance.

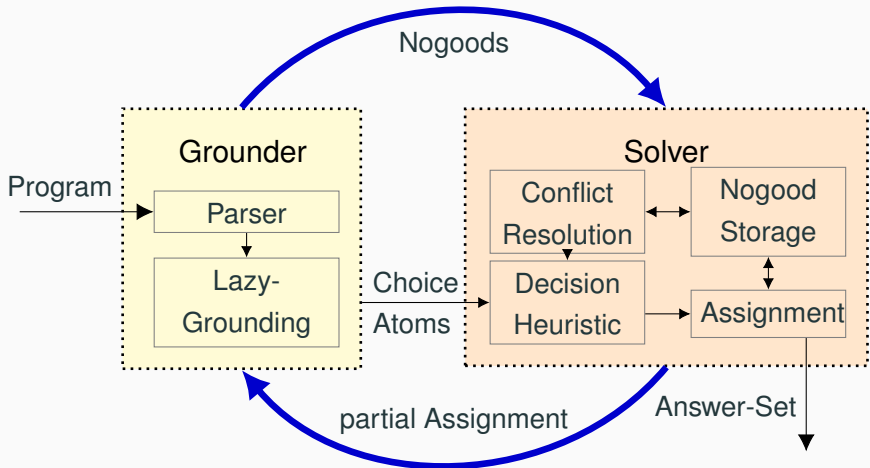
# The Alpha Solver

- Novel approach at lazy-grounding (first presented at LPNMR'17).
- Two components: **lazy-grounder** and **modified CDNL solver**.
- Blend **lazy-grounding** and state-of-the-art **CDNL-based search**.
- Represent rules by **nogoods** (similar to Clasp/ Clingo).
- **Loose-coupling** of lazy-grounding and search.
- **Goal:** Lazy-grounding with good search performance.

## Overview of Alpha

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# Architecture of Alpha



**Figure 1:** Alpha architecture, arrows indicate data flow.



## Representing Rules

- Constraint:  $\leftarrow sel(1), sel(2)$ .
- Nogood:  $ng = \{\mathbf{T}sel(1), \mathbf{T}sel(2)\}$ .
- Problem: distinguish  $\leftarrow not\ a, b$ . from  $a \leftarrow b$ .
- State-of-the-art solution, add constraint expressing:  
    *“If  $a$  is true, there must be a rule firing with  $a$  in its head.”*
- Lazy-grounding cannot know all rules with  $a$  in the head:  
     $a \leftarrow p(X)$ .
- Solution in Alpha:
  - Nogoods may have head and third truth value *must-be-true*.
  - Propagate: to *true* if head of nogood is implied, otherwise to *must-be-true* (and *false*).
  - Nogood  $\{\mathbf{F}a, \mathbf{T}b\}$  distinct from  $\{\mathbf{F}a, \mathbf{T}b\}_1$  with head.

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## Representing Rules by Nogoods

- Represent body of grounded rule  $r\sigma$  with fresh atom  $\beta(r, \sigma)$ .

### Example

- Let  $r$  be  $sel(X) \leftarrow dom(X), not nsel(X)$ .
- Given assignment  $A$  with  $dom(3)$  true.
- Grounder generates substitution  $\sigma : X \mapsto 3$ .
- Body representing atom:  $\beta(r, 3)$ .
- Generated nogoods:

$$n_1 : \{ \underline{\mathbf{F}\beta(r, 3)}, \mathbf{T}dom(3), \mathbf{F}nsel(3) \}_1 \quad n_2 : \{ \mathbf{T}\beta(r, 3), \mathbf{F}dom(3) \}$$

$$n_3 : \{ \mathbf{T}\beta(r, 3), \mathbf{T}dom(3) \} \quad n_4 : \{ \underline{\mathbf{F}sel(3)}, \mathbf{T}\beta(r, 3) \}_1$$

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# Core Alpha Algorithm

**Input:** A (non-ground) program  $P$ .

**Output:** The answer-sets  $\mathcal{AS}(P)$  of  $P$ .

Initialize  $\mathcal{AS} = \emptyset$ , assignment  $A$ , and nogood storage  $\Delta$ .

Run lazy grounder, obtain initial nogoods  $\Delta$  from facts.

**while** *search space not exhausted* **do**

    Propagate on  $\Delta$  extending  $A$ .

**if** *there exists conflicting nogood* **then**

        Analyze conflict, learn new nogood, and backjump. (a)

**else if** *propagation extended A* **then**

        Run lazy grounder wrt.  $A$  and extend  $\Delta$ . (b)

**else if** *exists an applicable rule* **then** (c)

        Guess as chosen by heuristic.

**else if** *exists unassigned atom* **then** (d)

        Assign all unassigned atoms to false.

**else if** *no atom in A assigned to must-be-true* **then** (e)

$\mathcal{AS} \leftarrow \mathcal{AS} \cup \{A\}$

        Add enumeration nogood and backtrack.

**else** (f)

        Backtrack.

**return**  $\mathcal{AS}$

## Propagation and 3WL

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# Propagation I

- Assignment  $A$  over  $F, T, M$ .

## Example

Nogood  $\delta = \{Fb, Tc, Td\}$

$A = (Fb, Tc)$  implies  $Fd$ .

- All but one literal  $s$  of  $\delta$  assigned the same value in  $A$ :  $\delta$  is **unit**  
 $\Rightarrow$  Extend  $A$  with **inverse** of  $s$ .
- Unit-propagation on nogood  $\delta$  two forms:
  - $\delta$  **weakly-unit** for  $s$  if  $\delta \setminus A^B = \{s\}$  and  $\bar{s} \notin A^B$ ,  
 $\Rightarrow$  propagation to *must-be-true* and *false*.
  - $\delta$  **strongly-unit** for  $s$  if  $\delta$  is nogood with head,  
 $\delta \setminus A = \{s\}$ ,  $hd(\delta) = s = Fa$ , and  $Ta \notin A$ ,  
 $\Rightarrow$  propagation to *true*.



# Propagation I

- Assignment  $A$  over  $\mathbf{F}, \mathbf{T}, \mathbf{M}$ . Boolean-projection  $A^{\mathcal{B}}$  maps  $\mathbf{M}$  to  $\mathbf{T}$ .

## Example

Nogood  $\delta = \{\mathbf{F}b, \mathbf{T}c, \mathbf{T}d\}$

$A = (\mathbf{F}b, \mathbf{T}c)$  implies  $\mathbf{F}d$ .

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 $\Rightarrow$  propagation to *must-be-true* and *false*.
  - $\delta$  **strongly-unit** for  $s$  if  $\delta$  is nogood with head,  
 $\delta \setminus A = \{s\}$ ,  $hd(\delta) = s = \mathbf{F}a$ , and  $\mathbf{T}a \notin A$ ,  
 $\Rightarrow$  propagation to *true*.

## Propagation II

- Propagation: Given assignment  $A$ , nogoods  $\Delta$ , find all  $\delta \in \Delta$  that are **weakly-unit** and **strongly-unit** under  $A$ .

### Definition

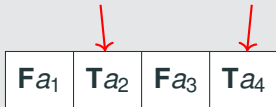
Let the *immediate unit-propagation*  $\Gamma_{\Delta}(A)$ , for set  $\Delta$  of nogoods and assignment  $A$  be:

$$\begin{aligned}\Gamma_{\Delta}(A) = & A \cup \{\mathbf{T}a \mid \exists \delta \in \Delta, \delta \text{ is strongly-unit under } A \text{ for } s = \mathbf{F}a\} \\ & \cup \{\mathbf{M}a \mid \exists \delta \in \Delta, \delta \text{ is weakly-unit under } A \text{ for } s = \mathbf{F}a\} \\ & \cup \{\mathbf{F}a \mid \exists \delta \in \Delta, \delta \text{ is weakly-unit under } A \text{ for } s = \mathbf{T}a\}\end{aligned}$$

Propagation is its least fixpoint, i.e.,  $\text{propagate}(A) = \text{lfp}(\Gamma_{\Delta}(A))$ .

# Watched Literals

## Example

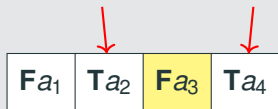


$$A_1 = \emptyset$$

1. Watched are two (arbitrary) unassigned literals in nogood.
  2. No watch on  $Fa_3$ , nothing to do.
  3. Watched literal assigned, move watch.
  4. Watch is moved, pointing at unassigned atom again.
  5. Only one literal unassigned, nogood is unit. Assign  $Fa_4$ .
    - Nogood with head, strongly-unit and weakly-unit?
- ⇒ Second set of watches; only 3 watches per nogood: **3WL**.

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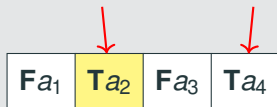


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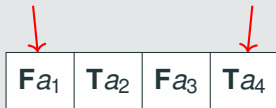


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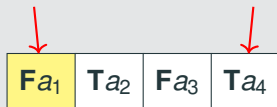


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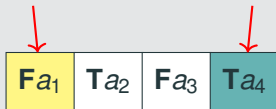


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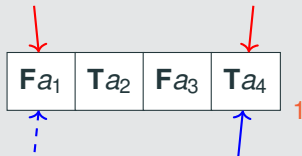
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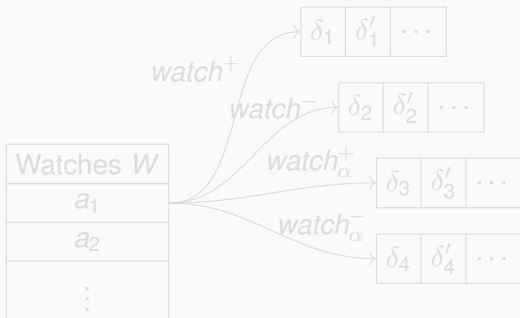
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# Watch Data Structure

## Definition

A *watch structure*  $W$  for assignment  $A$  and set of nogoods  $\Delta$  is mapping  $W : \mathcal{A} \rightarrow \Delta^4$  with

$$W(a) = (\text{watch}^+(a), \text{watch}^-(a), \text{watch}_\alpha^+(a), \text{watch}_\alpha^-(a)).$$

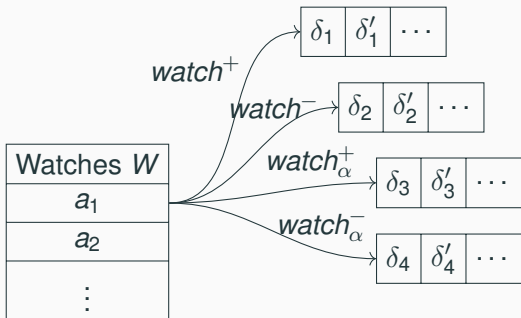


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## Watch Structure Invariants

- Assignment **grows** (choices, propagation), and **shrinks** (backtracking, backjumping).
- Shrinking shall not cause watches to be moved.
- Intuitively: watched literals must be unassigned (growing) or if nogood no longer satisfied, watched literals must be unassigned (shrinking).
- The watches of a nogood point at either
  1. two unassigned literals, or
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# Candidates for Watches

## Definition (Ordinary Watches)

For nogood  $\delta$  and assignment  $A$ ,  $s, s' \in \delta$  are *potential watches* if:

- (i)  $at(s)$  and  $at(s')$  both unassigned in  $A$ ; or
- (ii)  $at(s)$  complementary assigned, i.e.,  $\bar{s} \in A^B$ ,  
and either  $s'$  unassigned in  $A$  or  $dl^w(A, at(s')) \geq dl^w(A, at(s))$ .

## Definition (New $\alpha$ -watch)

For nogood  $\delta$  with head, assignment  $A$ ,  $s_\alpha \in \delta$  with  $hd(\delta) \neq s_\alpha$  is *potential  $\alpha$ -watch* if:

- (i)  $at(s_\alpha)$  unassigned or assigned to *must-be-true* in  $A$ , or  $s_\alpha$  complementary assigned in  $A$ ; or
- (ii) head is *true*, i.e.,  $\mathbf{T}at(hd(\delta)) \in A$  and  
 $dl^s(A, at(s_\alpha)) \geq dl^s(A, at(hd(\delta)))$ .

# Consistent Watch Structure

## Example

Consider  $A = \{\mathbf{Mc}, \mathbf{Fd}\}$  with  $dl^W(A, c) \leq dl^W(A, d)$ .

Nogood	Potential watches	Potential $\alpha$ -watches
$\delta_1 = \{\mathbf{Fa}, \mathbf{Tb}, \mathbf{Tc}, \mathbf{Fd}, \mathbf{Fe}\}_1$	$\mathbf{Fa}, \mathbf{Tb}, \mathbf{Fe}$	$\mathbf{Tb}, \mathbf{Tc}, \mathbf{Fe}$
$\delta_2 = \{\mathbf{Fc}, \mathbf{Fd}\}$	$\mathbf{Fc}, \mathbf{Fd}$	–
$\delta_3 = \{\mathbf{Fa}, \mathbf{Tc}\}_1$	–	$\mathbf{Tc}$

- Watch structure  $W$  **consistent** for assignment  $A$  and nogoods  $\Delta$ , if each  $\delta \in \Delta$  watched by potential watches  $s, s'$  and a potential  $\alpha$ -watch  $s_\alpha$  if  $\delta$  has head.

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# 3WL Propagation

**Input:** Assignment  $A$ , new assignments  $\Sigma$ , watches  $W$ , nogoods  $\Delta$ .

**Output:** Assignment  $A'$  or pair of  $A'$  and violated nogood  $d$ .

$A' \leftarrow A$

**while**  $\Sigma \neq \emptyset$  **do**

$\Sigma \leftarrow \Sigma \setminus \{Xa\}$  for some  $Xa \in \Sigma$ . // Process new assignment.

$(\Delta, \Delta_\alpha) \leftarrow \begin{cases} (\text{watch}^+(a), \text{watch}_\alpha^+(a)) & \text{if } X \in \{\mathbf{T}, \mathbf{M}\}, \text{ and} \\ (\text{watch}^-(a), \text{watch}_\alpha^-(a)) & \text{otherwise.} \end{cases}$

**foreach**  $\delta \in \Delta$  **do** // Propagation to  $\mathbf{M}, \mathbf{F}$ .

**if**  $\delta$  is violated **then**

**return**  $(A', \delta)$

**else if**  $\delta$  is weakly-unit for  $s$  **then**

            Let  $s' = \mathbf{M}b$  if  $s = \mathbf{F}b$  and  $s' = \mathbf{F}b$  otherwise.

$(A', \Sigma) \leftarrow (A \cup \{s'\}, \Sigma \cup \{s'\})$

        Remove old watches from  $\Delta$ , find new watches  $s, s'$ .

$\text{watch}(at(s)) \leftarrow \text{watch}(at(s)) \cup \{\delta\}$ ,  $\text{watch}(at(s')) \leftarrow \text{watch}(at(s')) \cup \{\delta\}$

**foreach**  $\delta \in \Delta_\alpha$  **do** // Propagation to  $\mathbf{T}$ .

**if**  $\delta$  is strongly-unit **then**

$(A', \Sigma) \leftarrow (A \cup \{\mathbf{T}at(\text{hd}(\delta))\}, \Sigma \cup \{\mathbf{T}at(\text{hd}(\delta))\})$

        Remove old watches from  $\Delta_\alpha$ , find new watch  $s$ .

$\text{watch}_\alpha(at(s)) \leftarrow \text{watch}_\alpha(at(s)) \cup \{\delta\}$

**return**  $A'$

One can show:

### Proposition

*Let  $W$  be a watch structure  $W$  for a set of nogoods  $\Delta$  that is consistent with an assignment  $A$  and let  $A' \supseteq A$  be a larger assignment with  $\Sigma = A' \setminus A$ . Then, Algorithm 2 running on  $A, \Sigma$ , and  $W$  returns either*

- 1. a pair  $(A'', \delta)$  such that  $A''$  is a consequence of  $A'$  and  $\Delta$  and  $\delta \in \Delta$  is violated by  $A''$ , or*
- 2. an assignment  $A'' = \text{propagate}(A')$  and the modified watch structure is consistent with  $A''$  and  $\Delta$ .*

# Evaluation

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# Benchmarks

- Compare **Alpha** (git-commit: 037b3f9), **Omiga** (git-commit: a65421f), **AsPeRiX** (2.5), **Clingo** (5.2.0).
- Performed on Linux machine, with two 12-core AMD Opteron 6176 SE CPUs and 128 GB RAM.
- Timeout 300 sec, memory limit 8GB.
- Request 10 answer sets from each solver.
- Available: [www.kr.tuwien.ac.at/research/systems/alpha](http://www.kr.tuwien.ac.at/research/systems/alpha)

# Benchmark: Ground Explosion

## Example

$dom(1) \dots dom(20). \quad sel(X) \leftarrow dom(X), not nsel(X). \quad nsel(X) \leftarrow dom(X), not sel(X).$   
 $\leftarrow sel(Y), sel(X), X! = Y. \quad p(X_1, X_2, X_3, X_4, X_5, X_6) \leftarrow sel(X_1), sel(X_2), sel(X_3), sel(X_4), sel(X_5), sel(X_6).$

Instance size	Alpha	Omiga	AsPeRiX	Clingo
8	1.37(0)	0.42(0)	5.54(0)	1.74(0)
10	1.48(0)	0.43(0)	0.02(0)	7.00(0)
16	1.60(0)	0.51(0)	0.03(0)	145.28(0)
18	1.64(0)	0.45(0)	0.03(0)	memout(1)
20	1.83(0)	0.46(0)	0.05(0)	memout(1)
50	1.31(0)	0.53(0)	0.11(0)	memout(1)
100	1.60(0)	0.81(0)	0.21(0)	memout(1)
500	2.19(0)	1.41(0)	1.06(0)	memout(1)
1000	2.30(0)	1.66(0)	2.21(0)	memout(1)

**Table 1:** Instance size is elements in domain. Runtime in seconds.

## Benchmark: Graph-5-Colorability

- Similar to ASPCOMP encoding (average on 10 random instances).

Instance size	Alpha	Omiga	AsPeRiX	Clingo
10/40	1.41(0)	14.33(0)	31.10(1)	0.02(0)
20/80	1.53(0)	234.93(6)	128.79(4)	0.02(0)
30/120	1.59(0)	300.00(10)	230.23(7)	0.03(0)
40/160	2.54(0)	300.00(10)	217.17(7)	0.04(0)
50/200	2.31(0)	300.00(10)	300.00(10)	0.04(0)
100/400	4.24(0)	300.00(10)	300.00(10)	0.06(0)
400/1600	22.54(0)	300.00(10)	300.00(10)	0.45(0)
500/2000	33.85(0)	300.00(10)	300.00(10)	0.68(0)
750/3000	67.22(0)	300.00(10)	300.00(10)	1.46(0)
1000/4000	119.94(0)	300.00(10)	300.00(10)	2.66(0)

**Table 2:** Size is no. vertices / no. edges. Runtime in seconds.

## Benchmark: Graph-5-Colorability

- Similar to ASPCOMP encoding.
- Easier instances.

Instance size	Alpha	Omega	AsPeRiX	Clingo
10/40	1.41(0)	14.33(0)	31.10(1)	0.02(0)
20/80	1.53(0)	234.93(6)	128.79(4)	0.02(0)
50/50	1.88(0)	290.47(9)	0.24(0)	0.03(0)
50/100	2.05(0)	300.00(10)	0.45(0)	0.03(0)
50/200	2.31(0)	300.00(10)	300.00(10)	0.04(0)
50/300	74.39(2)	300.00(10)	300.00(10)	0.07(0)
50/400	253.80(8)	300.00(10)	300.00(10)	0.06(0)
50/500	168.76(4)	300.00(10)	300.00(10)	0.04(0)

**Table 3:** Size is no. vertices / no. edges. Runtime in seconds.

## Benchmark: Cutedge

- Reachability after one edge removed from random graph.

Instance size	Alpha	Omiga	AsPeRiX	Clingo
100/30	12.59(0)	4.25(0)	0.78(0)	27.64(0)
100/50	11.87(0)	6.22(0)	1.79(0)	79.50(0)
200/30	22.90(0)	13.46(0)	13.29(0)	300.00(10)
200/50	45.95(0)	24.20(0)	35.18(0)	300.00(10)
300/10	16.92(0)	10.08(0)	8.54(0)	291.35(4)
300/30	59.58(0)	32.36(0)	72.09(0)	300.00(10)
500/10	62.46(0)	32.01(0)	70.38(0)	300.00(10)
500/30	300.00(10)	122.16(0)	300.00(10)	300.00(10)
400/10	40.97(0)	20.72(0)	27.61(0)	300.00(10)
400/30	300.00(10)	84.73(0)	284.71(4)	300.00(10)

**Table 4:** Size is no. vertices / percentage each edge present. Runtime in sec. 21



## Benchmark: Reachability

- Reachability on random graph (positive program).

Instance size	Alpha	Omiga	AsPeRiX	Clingo
1000/4	2.13(0)	1.21(0)	0.77(0)	0.11(0)
1000/8	3.19(0)	1.63(0)	2.57(0)	0.21(0)
10000/2	10.95(0)	7.82(0)	31.11(0)	0.52(0)
10000/4	13.06(0)	22.55(0)	130.00(0)	1.09(0)
10000/8	16.62(0)	56.93(0)	300.00(10)	2.27(0)

**Table 5:** Size is no. vertices / multiplier edges. Runtime in seconds.

Alpha is:

- **on-par** with other lazy-grounding solvers for **grounding** problems.
- **much faster** than other lazy-grounding solvers for **search** problems.
- slower than Clingo on search problems.
- not fastest, but already good **compromise**, especially if grounding problematic.
  
- Alpha freely available at:

<https://github.com/alpha-asp/Alpha>

## Conclusion

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## Related Work

- **Alpha** logical successor of **Omiga** (Dao-Tran et al., 2012):  
lazy-grounding (Rete), no efficient search.
- **Computation sequence** on GPUs (Dovier et al., 2016):  
nogoods-based, ground-and-solve, loop-nogoods etc. like Clasp.
- **AsPeRiX** (Lefèvre et al., 2017):  
lazy-grounding, no efficient search.
- More distanced:
  - **s(ASP)** (Marple et al., 2016):  
query-driven stable models, Prolog-like.
  - **Lazy-MX** (de Cat et al., 2015):  
lazy model expansion for FO(ID), no loops over negation, no ASP.

# Conclusion

- Alpha combines **lazy-grounding** and **CDNL search**.
- Efficient propagation techniques **3WL** for *must-be-true* and two forms of propagation.
- Benchmarks: Alpha effective where **grounding and solving** an issue.
- First lazy-grounding ASP solver for non-trivial search problems.
- Open issues / future work:
  - Using dependency information (e.g., SCCs) for faster solving.
  - Optimizations: forgetting, nogood minimization, stratified propagation.
  - Heuristics for grounding.

Thank you.

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